**D. (Advanced Monte Carlo)**

a) Two functions for calculating SD and SE are defined and declared in the SDandSE.hpp file.

b) The following table shows SDs and SEs as a function of NT and NSIM for Batch 1.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| NSIM  NT | 102 | 103 | 104 | 105 | 106 | 107 |
| 10 | C = 2.38525  P = 6.02450 | C = 2.11159  P = 6.07324 | C = 2.09379  P = 5.82172 | C = 2.10859  P = 5.84487 | C = 2.11637  P = 5.84018 | C = 2.11916  P = 5.83518 |
| Call SD = 5.68943  Call SE = 0.56894 | Call SD = 4.56751  Call SE = 0.14444 | Call SD = 4.38781  Call SE = 0.04388 | Call SD = 4.43197  Call SE = 0.01402 | Call SD = 4.44504  Call SE = 0.00445 | Call SD = 4.45136  Call SE = 0.00141 |
| Put SD = 6.34635  Put SE = 0.63463 | Put SD = 6.17285  Put SE = 0.19520 | Put SD = 6.05003  Put SE = 0.06050 | Put SD = 6.07578  Put SE = 0.01921 | Put SD = 6.08472  Put SE = 0.00608 | Put SD = 6.08158  Put SE = 0.00192 |
| 100 | C = 2.00892  P = 6.54862 | C = 2.14877  P = 5.88702 | C = 2.10932  P = 5.88443 | C = 2.13295  P = 5.87260 | C = 2.13288  P = 5.85106 | C = 2.13280  P = 5.84255 |
| Call SD = 4.21343  Call SE = 0.42134 | Call SD = 4.48775  Call SE = 0.14192 | Call SD = 4.47478  Call SE = 0.04475 | Call SD = 4.53000  Call SE = 0.01433 | Call SD = 4.52151  Call SE = 0.00452 | Call SD = 4.51104  Call SE = 0.00143 |
| Put SD = 6.57275  Put SE = 0.65727 | Put SD = 6.06444  Put SE = 0.19177 | Put SD = 6.06949  Put SE = 0.06069 | Put SD = 6.07197  Put SE = 0.01920 | Put SD = 6.05452  Put SE = 0.00605 | Put SD = 6.04795  Put SE = 0.00191 |
| 500 | C = 2.08171  P = 5.88529 | C = 2.17492  P = 6.13859 | C = 2.12329  P = 5.94285 | C = 2.14885  P = 5.83729 | C = 2.13249  P = 5.84333 | C = 2.13413  P = 5.84580 |
| Call SD = 4.23565  Call SE = 0.42356 | Call SD = 4.69434  Call SE = 0.14845 | Call SD = 4.49627  Call SE = 0.04496 | Call SD = 4.55679  Call SE = 0.01441 | Call SD = 4.51633  Call SE = 0.00452 | Call SD = 4.51710  Call SE = 0.00143 |
| Put SD = 6.19692  Put SE = 0.61969 | Put SD = 6.10368  Put SE = 0.19302 | Put SD = 6.08976  Put SE = 0.06090 | Put SD = 6.04876  Put SE = 0.01913 | Put SD = 6.04807  Put SE = 0.00605 | Put SD = 6.04719  Put SE = 0.00191 |
| 1000 | C = 2.36385  P = 6.14841 | C = 2.10416  P = 6.02480 | C = 2.12827  P = 5.93043 | C = 2.13874  P = 5.86757 | C = 2.13658  P = 5.83817 | C = 2.13320  P = 5.84573 |
| Call SD = 4.36268  Call SE = 0.43627 | Call SD = 4.54809  Call SE = 0.14382 | Call SD = 4.55583  Call SE = 0.04556 | Call SD = 4.55622  Call SE = 0.01441 | Call SD = 4.51429  Call SE = 0.00451 | Call SD = 4.51605  Call SE = 0.00143 |
| Put SD = 6.09495  Put SE = 0.60949 | Put SD = 5.92219  Put SE = 0.18728 | Put SD = 6.08910  Put SE = 0.06089 | Put SD = 6.05476  Put SE = 0.01915 | Put SD = 6.04834  Put SE = 0.00605 | Put SD = 6.04705  Put SE = 0.00191 |

Exact solution of Batch 1 is C = 2.13293, P = 5.84584. SDs for both P and C approach certain values (Call SD 🡪 4.51\*, Put SD 🡪 6.04\*) and SEs decrease to 0.001\* as NT and NSIM increase. As shown by the equation SE = SD/sqrt(NSIM), SE is proportional to the inverse of the square root of NSIM. This is true as shown by the statistics in the table. For example, when NT = 1000, Call SE = 0.4\* for NSIM = 100, and Call SE = 0.04\* for NSIM = 10000. To achieve a 2 decimal accuracy, SE for C has to be as small as 0.01, while SE for P has be as small as 0.001.

Exact solution of Batch 2 is C = 7.96632, P = 7.96632. An accuracy to two places behind the decimal point is achieved for P when NT = 100 and NSIM = 250000 and SE is slightly larger than 0.02, while the accuracy for C can only be achieved when NT = 100 and NSIM = 900000 and SE is about 0.01.